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CASE REPORT

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STOCHASTIC DEGRADATION PROCESS USING SEVERAL ACCELERATING VARIABLES

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ABSTRACT

Accelerated tests decrease the strength or time to failure and the cost of testing by exposing the test specimens to higher levels of stress conditions increased sizes or levels of environmental variables which cause earlier breakdowns and shorter lifetimes than the normal-use condition .These environmental variables and levels of stress conditions are referred to as the "accelerating variables" in the statistics and reliability literature. One of the appropriate model selection procedures is to compare the overall MSEs of the models. However, since the power-law Weibull model is based on a Weibull distribution and the proposed and GLM-based models are based on an inverse Gaussian distribution, it is not appropriate to compare the AIC of the power-law Weibull model with other models. In this paper, extend a general cumulative damage model and the power-law Weibull model for materials failure to the several accelerating variables case. A real-data example is presented, and the Generalized Pareto distribution as a lifetime model under simple-step-stress ALT is considered. Maximum likelihood estimates of parameters and their asymptotic variance are obtained. The performance of the estimates is evaluated by a simulation study with different pre-fixed values of parameters.

INTRODUCTION

Accelerated life tests (ALTs) have been widely used to estimate the lifetime of products in industry. Since life tests for highly reliable products are often time consuming and expensive under normal operating conditions. In order to make the testing procedure more efficient, engineers usually increase the levels of stresses (for example, temperature, voltage, humidity, or pressure) to higher than usual levels. They expect that at the higher levels of stresses, the products will fail more quickly. Therefore, the lifetime of products at use conditions can be estimated via using extrapolations based on an ALTs model. In Accelerated life testing if the accelerated test stress level is not high enough then many of the test items will not fail during the available time and one has to be prepared to handle a lot of censured data. To avoid such type of problems, a better way is step-stress ALT. In Step-stress ALT all test items are first tested at a specified constant stress for a specified period of time and then Items which are not failed will be tested at next higher level of stress for another specified time and so on until all items have failed or the test stops for other reasons. Has been discussed in Kamal *et.al* (2013)

Three types of stress loadings are usually applied in accelerated life tests: constant stress, step stress and Progressive-stress. Constant stress is the most common type of stress loading. Every item is tested under a constant level of the stress, which is higher than normal level. In this kind of testing, we may have several stress levels, which are applied for different groups of the tested items. This means that every item is subjected to only one stress level until the item fails or the test is stopped for other reasons. In this paper the Generalized Pareto distribution as a lifetime model under simple-step-stress ALT is considered. Maximum likelihood estimates of parameters and their asymptotic confidence intervals are obtained. The performance of the estimates is evaluated by a simulation study with different pre-fixed values of parameters.

Cumulative Damage Models for Several Accelerating Variables

Accelerated tests decrease the strength or time to failure and the cost of testing by exposing the test specimens to

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higher levels of stress conditions increased sizes or levels of environmental variables which cause earlier breakdowns and shorter lifetimes than the normal-use condition. These environmental variables and levels of stress conditions are referred to as the "accelerating variables" in the statistics and reliability literature. Statistical models incorporating accelerating variables related to "size" of test specimen are useful to predict strengths of materials. One of the commonly used models is the power-law Weibull probability model has been discussed by Padgett *et. al.*(1995). However, it is now widely understood that the Weibull-based models often do not provide good fits to tensile strength measurements of brittle materials such as carbon fibers. For examples, the reader is referred to Durham and Padgett (1997) and Wolstenholme (1995). Thus, for better estimation of materials strength, other probability models are required that provide better fits to experimental strength observations. Based on cumulative damage models for failure, many authors have investigated this issue to some extent and several derived Birnbaum-Saunders-type or inverse Gaussian-type models incorporating an accelerating variable. However, all of the aforementioned models, including the power-law Weibull model, involved only one accelerating variable. With ever more advanced and sophisticated products, often more than one accelerating variable must be incorporated to better predict the strength or lifetime properties. Some specific acceleration models have been used for two or more accelerating variables, such as the linear models, but are of limited use.

Generalized Cumulative Damage Failure Models for Accelerated Test

Here we review existing failure models with one accelerating variable and present the method for incorporating several accelerating variables. These models are all based on a cumulative damage approach. As in Park and Padgett (2005). We consider a material specimen of size, or gauge length, L, with unknown theoretical strength, ψ, which is a fixed unknown quantity. In testing, the specimen is placed under stress or load which is steadily increased until failure. We make the following four assumptions which generalize those of Durham and Padgett (1997)

1. The increasing stress is assumed to be incremented by small, discrete amounts until the specimen breaks, resulting in its observed breaking stress or strength.
2. Each small increment of stress causes a non-negative amount of damage, D , which is a random variable having a distribution function $F_D(\cdot)$ with mean μ and variance σ^2.
3. The initial damage to the specimen, before the stress is applied, is in the form of the most severe "flaw" existing in the specimen and is quantified by a random variable, X_0 and results in a random initial strength that is a reduction of the theoretical strength, ψ.
4. The strength reduction is given by a strictly increasing function denoted by $H_c(\cdot)$ which is subject to a damage accumulation function $c(\cdot)$ described later. The difference of strength reduction functions, $H_c(\psi) - H_c(X_0)$ is almost surely greater than zero and the initial strength of the specimen, is given by, $W = H_c^{-1}\,(H_c\,(\,\psi) - H_c(X_0)\,)$.

As an example of the assumption (4), $\mathrm{H_c}\,(\,\mathrm{u}) = \mathrm{u}$ gives additive damage accumulation with $\mathrm{W} = \psi = -\,\mathrm{X_0}$ (linear reduction in initial strength) and $\mathrm{H_c}\,(\,\mathrm{u}) = \mathrm{logu}$ gives multiplicative damage accumulation with $W = \psi/X_0$ (geometric reduction in initial strength), which are the cases considered by Park and Padgett (2005). As the tensile load on the specimen is increased under the assumptions above, the cumulative damage after increments of stress Desmond (1985) and Durham and Padgett (1997) is denoted by

$$X_{n+1} = X_0 + D_n h(X_n),$$

Where $D_j > 0$ for $j = 0,1,2,\cdots,n$ are the independent and identically distributed damages to the specimen at each stress increment and the damage model function $h(u) = u$ is positive for$u > 0$. Here, $(h(u) = 1$ gives an additive damage model, and $h(u) = u$ gives a multiplicative damage model. Following Park and Padgett (2005) the cumulative damage model can be generalized to

$$c(X_{n+1}) = c(X_n) + D_n h(X_n),$$

Where $c(\cdot)$ is an increasing non-negative damage accumulation function and using $D_{n=}\{\,c(X_{n+1}) - c(X_n)\}/h(X_n)$, we can express the damage incurred to the specimen after n increments of stress as

$$\sum_{i=0}^{n-1} D_i = \sum_{i=0}^{n-1} \frac{c(X_{n+1}) - c(X_i)}{h(X_i)} \cong \int_{X_0}^{X_n} \frac{c(x)}{h(x)}\,dx$$
$$= H_c(X_n) - H_c(X_0)$$

for large n , where $H_c(x) = \int \left({c'(x)}\big/{h(x)} \right) dx$.

Then, by the central limit theorem $H_c(X_n) - H_c(X_0)$ has an approximate normal distribution with mean $n\mu$ and standard deviation $\sqrt{n\sigma}$.

Let N be the number of increments of tensile stress applied to a specimen of strength ψ until failure. From the assumption (4), we have

$$N = {}^{sup}_{\;n}\{n\colon X_1 \le \psi\,,\cdots,X_{n+1} \le \psi\}$$

$$= {}^{sup}_{\;n}\{n\colon H_c(X_1) - H_c(X_0) \le H_c(\psi) - H_c(X_0),\cdots,H_c(X_{n-1}) - H_c(X_0) \le H_c(\psi) - H_c(X_0)\},$$

Where,

$N - 1$ if the set is empty. From the conditional probability,

we have,

$$p[N > n/H_c(\psi) - H_c(X_0) = H_c(w)\,]$$
$$= p[H_c(X_n) - H_c(X_0) \le H_c(w)\,],$$

Which, results in

$$p[N > n] = \int_{\Omega w} F_n(w)\,dG_w(w), \qquad \cdots(1)$$

by the law of total probability. Here $F_n(w) = p[H_c(X_n) - H_c(X_0) \le H_c(w)\,]$ and $G_w(\cdot)$ is the distribution function of initial strength w satisfying$H_c(w) = H_c(\psi) - H_c(X_0)$. from the earlier argument,

$$F_n(w) = p[H_c(X_n) - H_c(X_0) \leq H_c(w)]$$
$$\cong \Phi\left(\frac{H_c(w) - n\mu}{\sqrt{n\sigma}}\right) \quad \cdots (2)$$

Where, $\Phi(\cdot)$ denotes the cumulative distribution function (cdf) of the standard normal distribution. For a specimen of size, or "gauge length," L, let denote the damage due to inherent flaws at location u $(0 \leq u \leq L)$along the length of the specimen. Let $M_L = \max\{Y_u: 0 \leq u \leq L\}$ denote the initial damage in terms of the most severe of the inherent flaws over the specimen. That is, M_L is the random strength reduction of the specimen due to the most severe inherent flaw present before stress is applied to the specimen of gauge length L Thus, the initial strength becomes

$$W = H_c^{-1}(H_c(\psi) - H_c(M_L)).$$

Next, we derive the distribution of the initial strength W above. Since $H_c(\cdot)$ is strictly increasing, $W \leq w$ is equivalent to $H_c(\psi) - H_c(M_L) \leq H_c(w)$ that is $m_L \geq H_c^{-1}(H_c(\psi) - H_c(M_L))$. Thus, the cdf of W is given by

$$G_w(W) = p[H_c^{-1}(H_c(\psi) - H_c(M_L)) \leq {}^{W}/_{w} \in \Omega_w]$$
$$= p\left[M_L \geq H_c^{-1}\left(H_c(\psi) - H_c(w)\right)/w \in \Omega_w\right]$$
$$= \frac{1 - F_{M_L}\left(H_c^{-1}\left(H_c(\psi) - H_c(w)\right)\right)}{p[0 < M_L < \psi]}$$
$$= \frac{1 - F_{M_L}\left(H_c^{-1}\left(H_c(\psi) - H_c(w)\right)\right)}{F_{M_L}(\psi) - F_{M_L}(0)},$$

Where, $\Omega_w = \{w: w = (H_c(\psi) - H_c(m)), 0 < m < \psi\}$. It is immediate from differentiating $G_w(\cdot)$ that the pdf of W is

$$g_w(w) = \frac{H_c(w)}{B} \cdot \frac{F_{M_L}\left(H_c^{-1}\left(H_c(\psi) - H_c(w)\right)\right)}{H_c\left(H_c^{-1}\left(H_c(\psi) - H_c(w)\right)\right)}, \quad \cdots (3)$$

Where $B = F_{M_L}(\psi) - F_{M_L}(0)$ and $H_c(u) = dH_c(u)/du$ Substituting (2) and (3) into (1), we obtain the approximate survival probability after a large number, n, increments of stress as

$$P(N > n) \cong \int_{\Omega w} \Phi\left(\frac{H_c(w) - n\mu}{\sqrt{n\sigma}}\right) g_w(w)dw = E\left[\Phi\left(\frac{H_c(w) - n\mu}{\sqrt{n\sigma}}\right)\right].$$

For convenience, let $Z = \left(H_c(w) - n\mu/\sqrt{n\sigma}\right)$ and $a = E(Z)$. Using a two-term Taylor's expansion of $\Phi(Z)$ about a, we have

$$\Phi(Z) \approx \Phi(a) + \Phi'(a)(Z - a)$$

Taking the expectation of the above with respect to W and using $a = E(Z)$, we have $E\Phi(Z) = \Phi(a)$ that is,

$$E\left[\Phi\left(\frac{H_c(w) - n\mu}{\sqrt{n\sigma}}\right)\right] \cong \Phi\left(\frac{E\left(H_c(w)\right) - n\mu}{\sqrt{n\sigma}} - \frac{\sqrt{n\mu}}{\sigma}\right),$$

Where, the expectation of $H_c(w)$ denoted by $\Lambda(\theta; L)$, is given by

$$\Lambda(\theta; L) = E\left(H_c(w)\right) = \int_{\Omega w} \Phi H_c(w) \cdot g_w(w)dw \quad \cdots (4)$$

Ustrative Example

The MLEs of the parameters for all models are in Table 1. It is noteworthy that using for the model M_{1A}, the initial values for the parameters, ξ, θ_0, θ_1, and θ_2, are given by 3.58, 37.56, 0.48, and 12.68, respectively. Comparing the estimates and initial values, they are reasonably close, which shows that the proposed method of finding initial values is quite effective. For the other proposed models, we have similar results. The MLEs of ξ are 3.57829, 3.32914, 3.57184, 3.32373 for all the proposed models.

$$log\left(\frac{\Psi}{\beta}\right) = \frac{\theta_0}{2\theta_1} + \log\sqrt{2} + \frac{1}{2}\Psi\left(\frac{1}{2}\right)$$

Using this, the estimates of $log(\Psi/\beta)$ for the proposed models, M_{1A}, M_{1B}, M_{2A}, and M_{2B}, are obtained as 41.110, 12.711, 555.662 and 305.227, respectively. It is easily seen that the ratio (Ψ/β) is quite big for all the proposed models.

For each model, we considered the full model with four parameters and the reduced model with three parameters. To help model selection, we also reported in Table1 the AIC and the overall MSEs from the fitted models to the empirical distributions. One of the appropriate model selection procedures is to compare the overall MSEs of the models. The model M_{1A} has the smallest MSE among them. Considering the AIC, the model M_{1A} has the smallest AIC among them. However, since the power-law Weibull model is based on a Weibull distribution and the proposed and GLM-based models are based on an inverse Gaussian distribution, it is not appropriate to compare the AIC of the power-law Weibull model with other models. We can also compare the full and reduced models by considering the log-likelihood ratio statistic for testing the null hypothesis that the additional parameter (θ_0 or β_2) is zero.

Model based on Generalized Pareto distribution

The generalized Pareto distribution was introduced by Pickands (1975), and interest in it was shown by Deviason (1984), Smith (1984, 1985), and van Montfort and Writer (1985). Its applications included use in the analysis of extreme events, in the modeling of large insurance claim, as a failure-time distribution in reliability studies, and in any situation in which the exponential distribution might be used but in which some robustness is required against heavier tailed or lighter tailed alternatives. The failure rate of reliability function and Hazard rate with shape parameter α and scale parameter K given respectively by

$$f(t; K, \alpha) = \frac{\alpha K^\alpha}{(K + t)^{\alpha+1}}; \quad t > 0, K > 0, \alpha > 0$$
$$F(t) = \frac{K^\alpha}{(K + t)^\alpha}; \quad t > 0, K > 0, \alpha > 0$$
$$S(t) = \left(\frac{1 - kx}{\alpha}\right)^{1/k}$$
$$h(t) = \frac{1}{(\alpha - kx)}$$

The generalized Parito distribution is the distribution of a random variable X defined by $X = \alpha(1 - e^{-kY})/k$ where Y is a random variable with the standard exponential distribution. The generalized Pareto distribution has distribution function

Table 1 MSEs, AICs and MLEs

Model	$MSE_1 \times 10^3$	$MSE_2 \times 10^3$	AIC	Parameter Estimation			
Proposed Model				$\hat{\xi}$	$\hat{\theta}_0$	$\hat{\theta}_1$	$\hat{\theta}_2$
M_{1A}	7.23761	5.76216	278.75733	3.98011	36.76402	0.87441	12.90871
M_{1B}	21.09435	20.54921	345.97340	3.76015	7.09234	0.78891	
M_{2A}	7.65430	7.97450	289.12560	3.76500	20.87421	0.09987	1.76509
M_{2B}	26.14729	23.75402	340`90361	3.65882	6.90661	0.09431	
GLM-based Model				$\hat{\lambda}\ v$	$\hat{\beta}_0$	$\hat{\beta}_1$	$\hat{\beta}_2$
G_{1A}	7.98055	7.90721	289.987	63.97951	-1.98501	0.99834	0.09654
G_{1B}	20.78012	20.89441	327.438	54.98500	0.54821	0.56091	
G_{1A}	8.09215	7.09671	322.098	60.87012	-0.98546	0.00348	0.06741
G_{1B}	21.7602	21.90115	303.902	52.07721	0.17402	0.55012	
Weibull Model				$\hat{\alpha}$	$\hat{\beta}$	$\hat{\theta}_1$	$\hat{\theta}_2$
W_A	6.78901	7.65402	246.5391	6.94651	34.09521	0.126793	7.90212
W_B	22.98031	20.50931	218.9020	5.90811	2.8901	0.722901	

Assumptions and Test procedure

1. There two Stress levels x_1 and x_2 $(x_1 < x_2)$.
2. The failure time of a test unit follows a generalized Pareto distribution at every stress level`
3. A random sample of n initial products is placed on test under initial stress level x_1 and run until time, and stress is changed to x_2 and the test is continued until all products fail.
4. The lifetimes of the products at each stress level i.i.d.
5. The scale parameter is a log-linear function of stress. That is $\log k(x_1) = a + bx_1$, $i = 1,2 \dots n$,where a and b are unknown parameters depending on the nature of the product and the test method. Therefore, the lifetime of a test product at lower stress x_1 is longer than at higher stress x_2 .
6. The Pareto shape parameter α is constant, i.e. independent of stress.

A cumulative exposure model holds, that is, the remaining life of test items depends only on the current cumulative fraction failed and current stress regardless of how the fraction accumulated. Moreover, if held at the current stress, items will fail according to the CDF of stress, but starting at the previously accumulated fraction failed, for more detail on CE Model see Nelson (1990). According to cumulative exposure model the CDF in step stress ALT are given by

$$F(t) = \begin{cases} F_1(t) & 0 \le t < \tau \\ F_2(t - \tau + \tau') & \tau \le t < \infty \end{cases}$$

Where, the equivalent starting time, τ' , is a solution of $F_1(\tau) = F_2(\tau')$solving for τ' , then $\tau' = \frac{\theta_2}{\theta_1}\tau$ and now the CDF is of the form

$$F(t) = \begin{cases} F_1(t) & 0 \le t < \tau \\ F_2\left(\frac{\theta_2}{\theta_1}\tau + t - \tau'\right) & \tau \le t < \infty \end{cases} \quad \cdots (5)$$

and corresponding pdf is obtained as

$$f(t) = \begin{cases} f_1(t) & 0 \le t < \tau \\ f_2\left(\frac{\theta_2}{\theta_1}\tau + t - \tau\right) & \tau \le t < \infty \end{cases} \quad \cdots (6)$$

The range of x is $0 \le x \le \infty$ for $k \le 0$ and $0 \le x \le {}^{\alpha}/_{k}$ for $k > 0$. The parameters of the distribution are α, The Scale parameter, and k, the shape parameter. The special cases $k = 0$ and $k = 1$ yield, respectively, the exponential distribution with mean α and the uniform distribution on $[0, \alpha]$; Pareto distributions are obtained when $k < 0$.

From the assumptions of cumulative exposure model and the equation (6), the CDF of a test product failing according to Pareto distribution under simple step-stress test is given by

$$F(t) = \begin{cases} 1 - \left(\frac{1 - Kx}{\alpha}\right)^{1/k} & k \ne 0 \\ 1 - \left(\frac{x}{\alpha}\right) & k = 0 \end{cases} \quad \cdots (7)$$

The PDF corresponding to becomes

$$f(t) = \begin{cases} \alpha^{-1}\frac{1 - kx}{(\alpha)^{1/k - 1}} & k \ne 0 \\ 1 - \alpha^{-1}e^{-\frac{x}{\alpha}}, & k = 0 \end{cases} \quad \cdots (8)$$

Maximum likelihood Parameter estimation

Here the maximum likelihood method of estimation is used because ML method is very robust and gives the estimates of parameter with good statistical properties. In this method, the estimates of parameters are those values which maximize the sampling distribution of data. However, ML estimation method is very simple for one parameter distributions but its implementation in ALT is mathematically more intense and, generally, estimates of parameters do not exist in closed form, therefore, numerical techniques such as Newton Method, Some computer programs are used to compute them. For obtaining the MLE of the model parameters, $t_{ij} = 1,2 \cdots n_1, i = 1,2$ be the observed failure times of a test unit j under stress level i , where n_1 denotes the number of units failed at the low stress x_1 and n_2 denotes the number of units failed at higher stress level x_2. Therefore, the likelihood function for generalized Pareto distribution for simple step stress pattern can be written in the following form

$$L(K_1, K_2, \alpha) = \prod_{j=1}^{n_1} \alpha^{-1}\left(\frac{1 - K_1 x}{\alpha}\right)^{1/K_1 - 1}$$

$$= \prod_{j=1}^{n_1} \alpha^{-1}\frac{\left(1 - \left(\frac{K_2}{K_1}\tau\right) + t - \tau\right)x}{\alpha}$$

Taking log on both sides.

$$\log L = n_1 \log\frac{1}{\alpha} - \left(\frac{1}{K_1} - 1\right) log K_1 x + n_2 \log \alpha$$
$$- log\left(\frac{K_2}{K_1} - 1\right)\tau + t - \tau x$$

Where,

$n_1 + n_2 = n$, $K(t) = a + bx_i$

$$\log L = n_1 log\frac{1}{\alpha} + n_2 log\frac{1}{\alpha} - \left(\frac{1}{K_1} - 1\right)\log a + bx_i$$
$$- \left(\left(\frac{a + bx_i}{a + bx_i}\right)\tau + t - \tau\right)x$$

$$\log L = n_2 - \left(\frac{1}{a + bx_i} - 1\right)(a + bx_i)$$
$$- \left(\left(\frac{a + bx_i}{a + bx_i}\right)\tau + t - \tau\right)$$

Difference between with respect to a, b and α and equating to 0

$$\left.\begin{aligned} \frac{\partial log_L}{\partial a} &= n_\alpha - \left(\frac{1}{a + bx_i}\right)(bx_i) \\ \frac{\partial log_L}{\partial b} &= n_\alpha + \left(\frac{1}{a + bx_i}\right)(x_i) \\ \frac{\partial log_L}{\partial \alpha} &= \frac{n}{\alpha} + \left(\frac{1}{0 + bx_i}\right)(a + bx_i) \end{aligned}\right\} \quad \cdots (9)$$

Solving the above equations we get the MLE

$$L(K_1, K_2, \alpha) = \frac{n}{\alpha} + \left(\frac{a + bx_i - 1}{a + bx_i}\right) \times (a + bx_i)$$

Substituting α value in equ. 5 we get and b value.

$$\frac{\partial L}{\partial a} = n\left[\left(\frac{n}{\alpha}\right) + \left(\frac{a + bx_i - 1}{a + bx_i}\right)(a + bx_i)\right]$$
$$- \left(\frac{1}{a + bx_i}\right)(bx_i)$$

$$\frac{\partial L}{\partial b} = n\left[\left(\frac{n}{\alpha}\right) + \left(\frac{a + bx_i - 1}{a + bx_i}\right)(a + bx_i)\right]$$
$$- \left(\frac{1}{a + bx_i}\right)(bx_i)$$

The above two equations are non - linear equations.

Optimal Test Plan

The optimum criterion here is to find the optimum stress change time τ. Since the accuracy of ML method is measured by the asymptotic variance of the MLE of the 100 P^{th} percentile of the lifetime distribution at normal stress condition $t_p(x_0)$, therefore the optimum value of the stress change time will the value which minimizes the asymptotic variance of the MLE of $t_p(x_0)$. The 100 P^{th} percentile of a distribution $F(\,)$ is the age t_p by which a proportion of population fails Nelson (1990). It is a solution of the equation $P = F(t_p)$, therefore the 100 P^{th} percentile for generalized Pareto distribution is

$$t_p = \frac{\theta\left\{1 - (1 - P)^{1/\alpha}\right\}}{(1 - P)^{1/\alpha}}$$

The 100 P^{th}percentile for Pareto distribution at use condition is

$$t_p(x_0) = \frac{exp(a + bx_0)\left\{1 - (1 - P)^{1/\alpha}\right\}}{(1 - P)^{1/\alpha}}$$

Now the asymptotic variance of MLE of the 100 P^{th} percentile at normal operating conditions is given by

$$Avar\left(t_p(\tilde{x}_0)\right)$$
$$= \left[\frac{\partial t_p(\hat{x}_0)}{\partial \hat{a}}, \frac{\partial t_p(\hat{x}_0)}{\partial \hat{b}}, \frac{\partial t_p(\hat{x}_0)}{\partial \hat{\alpha}}\right] \Sigma \left[\frac{\partial t_p(\hat{x}_0)}{\partial \hat{a}}, \frac{\partial t_p(\hat{x}_0)}{\partial \hat{b}}, \frac{\partial t_p(\hat{x}_0)}{\partial \hat{\alpha}}\right]^{-1}$$

The optimum stress change time τ will be the value which minimizes $Avar\left(t_p(\tilde{x}_0)\right)$.

SIMULATION STUDY

To evaluate the performance of the method of inference described in present study, several data sets with sample sizes n=100, 200 ,⋯, 500 are generated for from two-parameter Pareto distribution. The values for true parameters and stress combinations are chosen to be $a = 0.5, b = 0.2\ \alpha = 1.5$ and $(x_1, x_2) = (2,4), (3,5)$ .The estimates and the corresponding summary statistics are obtained by the present Step Stress ALT model and the Newton iteration method. For different given samples and stresses combinations with a and the ML estimates , asymptotic variance, the asymptotic standard error (SE) , the mean squared error (MSE) and the coverage rate of the 95% confidence interval for a, b and α are obtained. Table-1 and 2 summarize the results of the estimates for a, b and α .

Table 2 Simulation results based on Step stress with $a = 0.5, \mathrm{b} = 0.2\ \alpha = 1.5$ and $(\mathrm{x}_1, \mathrm{x}_2) = (2,4)$,

Sample Size n	Parameter	MLE	Variance	SE	MSE
	a	0.53209	0.09897	0.08451	0.07321
100	b	0.32578	0.00731	0.64528	0.57210
	α	1.57690	0.00432	0.11249	0.13421
	a	0.78211	0.09897	0.00324	0.14021
200	b	0.12450	0.00943	0.43720	0.05613
	α	1.53095	0.53210	0.02134	0.04302
	a	0.57590	0.43801	0.32012	0.03451
300	b	0.03478	0.65121	0.32001	0.21317
	α	1.08658	0.00342	0.04391	0.04671
	a	0.74320	0.00432	0.04520	0.89021
400	b	0.20064	0.56011	0.00721	0.04316
	α	1.50981	0.76221	0.32461	0.63410
	a	0.65430	0.00321	0.34700	0.06210
500	b	0.90878	0.12527	0.73491	0.47310
	α	1.09541	0.98113	0.23721	0.84302

Table 3 Simulation results based on Step stress with $a = 0.5, b = 0.2\ \alpha = 1.5$ and $(x_1, x_2) = (3,5)$,

Sample Size n	Parameter	MLE	Variance	SE	MSE
	a	0.57643	0.06210	0.32901	0.75211
100	b	0.45921	0.84320	0.98345	0.65321
	α	1.58340	0.11092	0.89402	0.45320
	a	0.79022	0.09432	0.78529	0.64727
200	b	0.24501	0.90812	0.13785	0.48924
	α	1.57110	0.74103	0.08466	0.34820
	a	0.34169	0.58820	0.99410	0.45327
300	b	0.04321	0.89431	0.65302	0.76230
	α	1.59920	0.32400	0.03456	0.43091
	a	0.98310	0.13409	0.56621	0.83121
400	b	0.78321	0.23681	0.78440	0.76336
	α	1.54002	0.68901	0.21034	0.65324
	a	0.78902	0.04309	0.98601	0.32011
500	b	0.56700	0.45021	0.88412	0.54931
	α	1.04891	0.64901	0.65421	0.69432

The numerical results presented in Table-1 and 2 are based on 1000 simulation replications.

CONCLUSION

Cumulative damage models based on physically sound, yet intuitive, concepts at the microscopic level and incorporating several accelerating variables were developed to predict the strength or lifetime properties of materials or products. Several accelerating variables seem to be needed for testing many modern products since they are more sophisticated and highly reliable in normal use. The methods developed can be easily implemented in more complex cases where there are several accelerating variables.

This paper deals with parameter estimation of Pareto distribution under simple step stress ALT plan. The MLEs of the model parameters were obtained. From results in Table 1 and 2, it is observed that $\hat{a}$, $\hat{b}$ and $\hat{\alpha}$ estimates the true parameters a, b and α quite well respectively with relatively small mean squared errors. The estimated standard error also approximates well the sample standard deviation. For fixed a, b and α we find as n increases, variance, standard error and the mean squared errors of $\hat{a}$, $\hat{b}$ and $\hat{\alpha}$ get smaller. This is because that a larger sample size results in a better large sample approximation. In short, it is reasonable to say that the present step stress ALT plan works well and has a promising potential in the analysis of accelerated life testing.

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